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Investigation 1 Projectile Motion

Planning

Hypothesis

The smaller the air resistance experienced by a projectile, the closer the launch angle is to the idealised launch angle for the maximum range of the projectile.

Support for the hypothesis

To alter air resistance, projectiles of different masses will be used. In an idealised world mass has no effect on range. However, with air resistance, lighter objects will experience greater deceleration given the same air resistance force. Heavy objects will experience a smaller deceleration due to air resistance, and so will have launch angles closer to the idealised launch angle. In addition, given the same amount of kinetic energy lighter projectiles travel faster (which increases the drag force - see research).

Variables

Independent: The air resistance of the projectile (altered by changing the mass)
Dependent: The launch angle required for the maximum range of the projectile

Controlled:

- The height at which the projectile is launched
- The specific design of launcher used
- The force applied to the projectile (as given by the distance the launcher is drawn back)
- The surface area of the projectile (as given by similarly sized projectiles)

Uncontrolled:

- The spin of the ball
- The velocity of the ball (cannot be controlled if force applied to the projectile is controlled)
- The way the ball bounces within the launcher during launch

Equipment list

- Plastic half tube ramp
- Ball launcher
- Rubber bands
- Projectiles (table tennis ball and golf ball)
  - m(Table tennis ball) = 2.6g
  - m(golf) = 45.7g
  - r(Table tennis ball) = 20.0 mm
  - r(golf) = 21.5mm
- Measuring tape
- Ruler

Experimental setup

Definition of axis:
x-axis - the movement parallel to the tape measure (the horizontal range)
y-axis - the movement perpendicular to the plane of the floor
z-axis - the movement perpendicular to the line of tape measure

See attached Fig. 2 for experimental setup diagram
See attached Fig. 3 for launcher design

Method

1. Set up equipment as shown in experimental setup

2. Align the point at which the launcher is at rest (i.e. not drawn back) with the pivot point (the corner of the chair; see diagram)
3. Adjust ramp for precise angle required by placing boxes beneath the ramp contact point
4. Draw back launcher to a marked point to fire (30 cm)
    1. Test both projectiles with the same drawback on launcher
    2. Test both projectiles 8 times at 30°, 45°, 60° different angles
5. If the launch is visibly off-course (i.e. collides with the sides of the launcher) , repeat the launch.
6. Fine tune launch angles to find optimal launch angle for each projectile
    1. Select launch angles close to the maximum range launch angle determined in 4. (i.e. 25°, 35°, 40°) and repeat

Note on optimal angle

Because the object is fired 0.7 m above ground level, the optimal angle for maximum range will not be 45° even in idealised conditions. Kenny (2010) showed that the optimal angle given for maximum range for a projectile above ground level is :

$$\sin\theta = \left(2\left(\frac{gh}{v^2}+1\right)\right)^{-1/2}$$

Generally, the higher up the projectile is above ground level, the lower the launch angle. If projectile speed is 5 m/s (a rough approximation of projectile speeds in this experiment) , the optimal angle is reduced to 38° for maximum range.This occurs as there is an interaction between maximising the time spent in the air and maximising the horizontal x-velocity.

Conducting

Please see attached tables for results. Generally, no significant outliers were discovered, and noticeably errant launches were repeated upon consensus of the group.

Analysis

Foreword on Uncertainty Calculations

Uncertainty was generally not possible to calculate formally. In this experiment, calculable uncertainty occurred in several areas: the calculation of the angle of launch, the measurement of the range of the projectile, the length to which the launcher was drawn back, amongst others.

For example, uncertainty on the calculation of angle of launch:

Uncertainty on ruler $= \pm 0.5cm$

For 45°:

$$\cos(\theta) = (/H)$$

$$\equiv \frac{(70\pm0.5)-(20\pm0.5)}{70\pm0.5}$$

$$Ua = \frac{50\pm1.0}{70\pm0.5}$$

$$Ur = \frac{50\pm1/20}{70\pm0.5/70}$$

$$= 0.714\pm0.057$$

$$Ua = 0.714\pm0.016$$

$$\theta_{\min} = 43.1°$$

$$\theta_{\max} = 45.7°$$

Thus there is +0.7, -1.9° asymmetrical uncertainty on the 45° launch angle, given the uncertainties on the calculation of the angle for launch.

However, it can be argued that most of the uncertainty occurred as a result of the inherent complexity of the experiment.The complicated factors affecting a launch would have included uncertainties impossible to formally calculate, resulting in a spread of results unaccounted for by the calculated uncertainties.

An approximation of the uncertainty associated with the launch could be calculating the standard deviation of the sample, and by making the assumption that the data is normally distributed the uncertainty would be given by 3 standard deviations, which would produce a 99.7% confidence interval (i.e. 99.7% of data points will fall in within 3 std dev of the mean). It should be noted that the sample size of data collected is small, potentially affecting the validity of this uncertainty calculation.

Calculating optimal launch angles

As discussed above, the launch angle can only be determined to the nearest degree at best, because beyond this level uncertainties in the determination of launch angle and the measurement of the range of the launch start to take effect. This means that some interpolation will be required to determine the optimal launch angle. However, optimisation indicates that the function of range with respect to launch angle cannot be a linear function, because there would be no maximum range. A parabolic function was fitted to the data, and because of the relatively good correlation coefficient it was determined that the parabolic fit was a good representation of the data.

Discussion of optimal launch angles

The optimal launch angles were calculated with raw data. See attached Fig. 6,7 for graphs.

An optimal launch angle was calculated by optimising the function visually, given the curve of best fit. Both graphs showed that the optimal launch angle was less than 45°, as the range of the projectile dropped noticeably at 45° and at 60°.

For the golf ball, the optimum angle was observed at $\sin(x)=0.51\pm0.005$, which is $31\pm0.33°$. For the table tennis ball, the optimum angle was observed at $\sin(x)=0.55\pm0.005$, which is $33\pm0.34°$ (all angles given to 2 SF). This was determined graphically, and the uncertainty values given are from the digital mouse pointer, which had an uncertainty of $\pm0.00005 \sin(x)$; only $\pm0.005 \sin(x)$ was claimed.

Notice that these values lie close enough together that uncertainty on the determination of the launch angle is now a significant factor. Thus it would appear that that there is no discernible effect of mass on optimum angle.

However, it should be noted that overall, the effect of air resistance on the golf ball was much less than that on the table tennis ball. From inspection of the graph, it can be seen that the distance travelled based on angle falls away much slower for the golf ball than the table tennis ball.

Conclusion

From the data, it is not possible to determine the effect different ball masses has on launch angle optimised for maximum range. However, as a general trend, the range of the golf ball was not as affected by launch angle, compared to the table tennis ball.

Evaluation

The rubber bands would twist after each firing, and remain in a twisted position. This was not noticed until part way through the experiment, at which point steps were taken to undo this twisting. This twisting could alter the tension and thus the force at which the rubber bands fired.

The rubber bands lost elasticity over time. This is due to the natural wear and tear of materials, and for practical reasons it was assumed that the elasticity of the rubber bands remained the same. It is likely that the rubber bands would have produced a smaller force as the experiment progressed.

The assumption was made that the launcher was considered to have fired the projectile once it had reached its position of rest. As such, this point was used as a pivot when changing angles for all projectiles in order to keep height constant. Although this point was approximately 15 cm from the end of the launch tube, the friction over this part was considered minimal as the tube modelled the flight path of the projectile reasonably accurately over short distances.

The projectile would experience deviation perpendicular to the intended flight path, due to the inherent limitations of the launcher. The deviations in angles were observed to be minimal (30 cm z-axis spread). It is known that for small deviations in an angle x, the effect on a trigonometric function cos(x) or sin(x) is less than proportional (this effect can be derived with application of Taylor series, see Fig 5 for derivation). According to the attached Fig. 4, the measured range is given by Acos(x), and so this means that the measured range will not differ significantly from the actual range of the object. This meant that the effect of the deviation in the z-axis direction on range could be ignored.

Although the 35° angle was intended to be tested, this did not occur due to a variety of reasons. There was insufficient time on the first day for it to be completed with the other results. However, when the launcher was retrieved for a the continuation of the experiment it was discovered that the elastic bands had snapped, and they had been replaced with different rubber bands. This meant that the force exerted on the projectiles would not be the same given the same displacement, and so any results produced would not be relevant. This was unfortunate as the results indicated that the maximum range could lie in the 30°-40° range.

Discussion

Air resistance will have varying effects on projectiles of varying mass, even if they have the same surface area, travel at the same velocity, travel through the same medium and have the same geometry. As it was not possible to control the velocity of the projectile, force was controlled instead.

Given the same force, the golf ball would have a lower launch velocity than the table tennis ball, because given the same kinetic energy, the lighter object would have a higher velocity. This means that the table tennis ball would experience a larger drag force (as $F_{drag} \propto v^2$). Given that the table tennis ball weighs ~ 1/20th that of the golf ball, it would experience at least 20 times the deceleration experienced by the golf ball (and probably more due to its higher launch velocity and subsequent larger drag force). It thus stands to reason that minimising the time spent in the air would minimise this deceleration, and this can be achieved by lower launch angles. This is supported by the experimental data demonstrating an optimal angle of launch at 33°.

Generally, the spin of the ball was minimised by the design of the launcher, so the Magnus effect was fairly minimal and was not seen to affect the experiment.

The golf ball did not travel far enough or fast enough for the dimples to have a significant effect on the flight path.

Air resistance Research

When projectiles move through air, they must exert a force to accelerate the air in front of their path to the velocity they are travelling at; this is because air is viscous. This force (known as the drag force) can be calculated experimentally via this equation (UNSW, 2005):

$$F_{Drag} = \frac{1}{2} C_D A \rho v^2$$

where:

$C_D =$ Drag coefficient

A = surface area of the projectile

$\rho =$ density of the fluid causing drag

$v =$ velocity

This shows that air resistance is related to the velocity of the projectile, the drag coefficient for that particular projectile (which itself depends on the geometry of the projectile), and the surface area of the projectile. In turn, the effect that air resistance has on an object is related to the mass of that object. This means that maximum range should be obtained where the work done by the drag force on the projectile($W = F_{Drag} s$) is be minimised, because this allows the projectile to maintain a higher level of energy and so a higher velocity, enabling greater range.

For launch angles greater than the idealised 45°, the projectile experiences a longer flight path. This allows the drag force to act on the projectile for a longer distance, which could reduce the maximum range. However, for launch angles less than 45°, the projectile would experience a larger component of the drag force along the axis of horizontal travel compared to angles greater than 45°, causing the projectile to slow down faster in the horizontal direction. In addition, projectiles fired at below 45° spend more time near their maximum velocity (if a idealised situation is temporarily imagined, the sub-45° projectile would have a larger initial x-axis velocity which does not change; compared to the super-45° projectile, which has a smaller initial x-axis velocity, the sub-45° projectile would have a higher average velocity). As the drag force is proportional to the square of the velocity this would cause more drag to be exerted on the projectile. This problem, however, cannot be analytically solved as it depends on variables too complex to calculate at this level.

The geometry of the projectile can affect the range. For example, golf balls are produced with dimples. This produces a turbulent boundary layer and reduces drag, allowing golf balls to travel further. Spherical objects are useful in that the drag coefficient (dependent on projectile geometry) and the surface area (A) would not change as the projectile tumbles through the air, meaning that the drag force must be proportional to the square of the velocity of the projectile.

Table tennis balls are well-known for their spin. The rotation of the ball creates a difference in pressures, causing a force to act towards the side of lower pressure. This is known as the Magnus effect, which can create a force perpendicular to the path of travel and so reduce or extend the range of the projectile, depending on which way it is spinning.

Figures

Figure 1
Effect of dimples on air resistance.
(Scott, 2005)

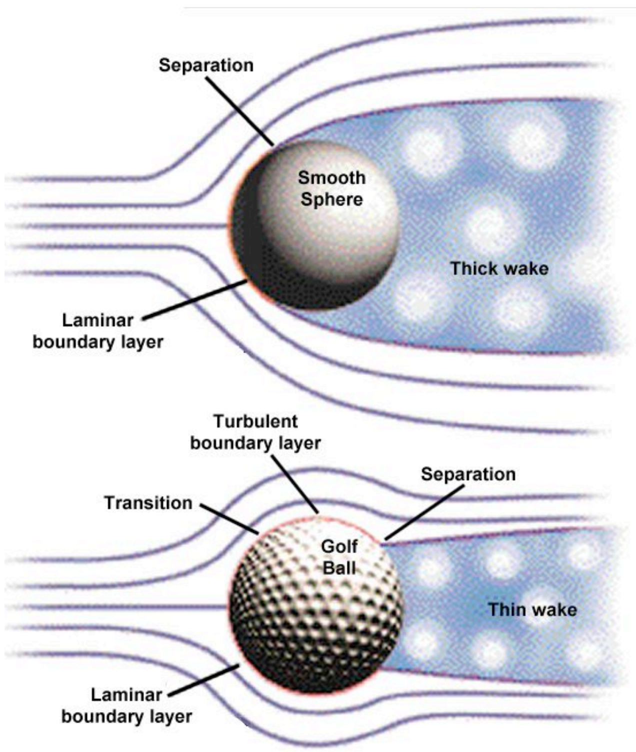


Figure 2
Launcher Diagram setup
(Lee, 2012)

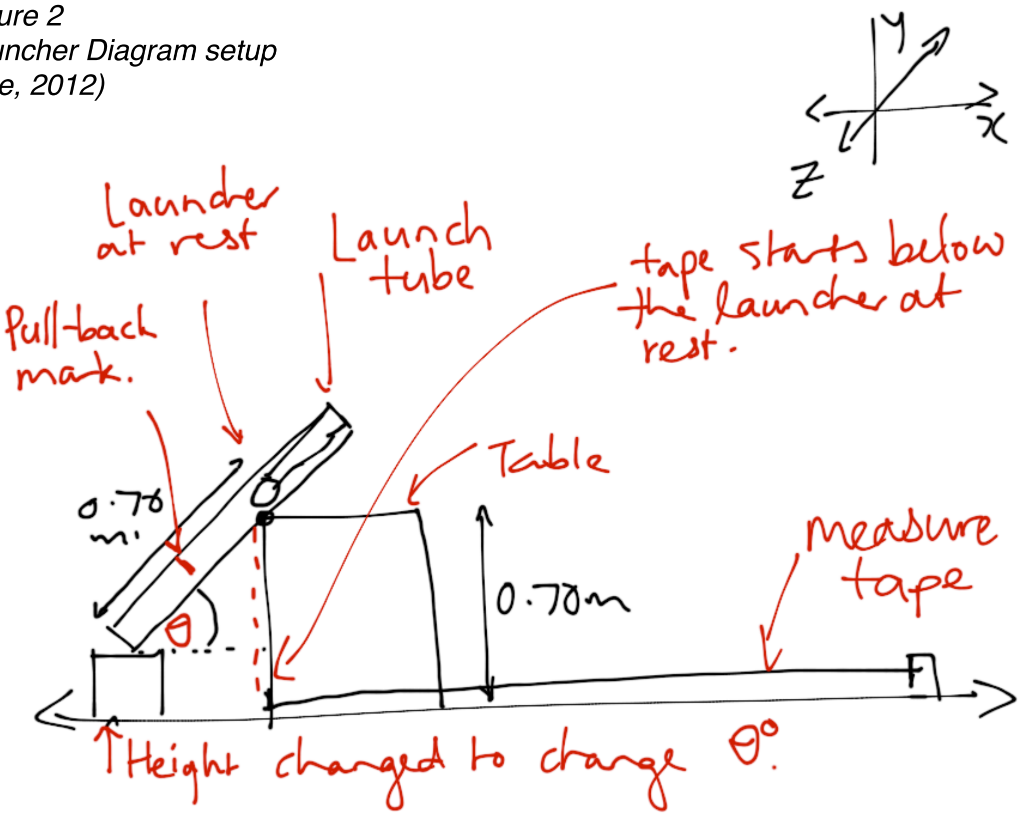


Figure 3
Launcher design
(Lee, 2012)

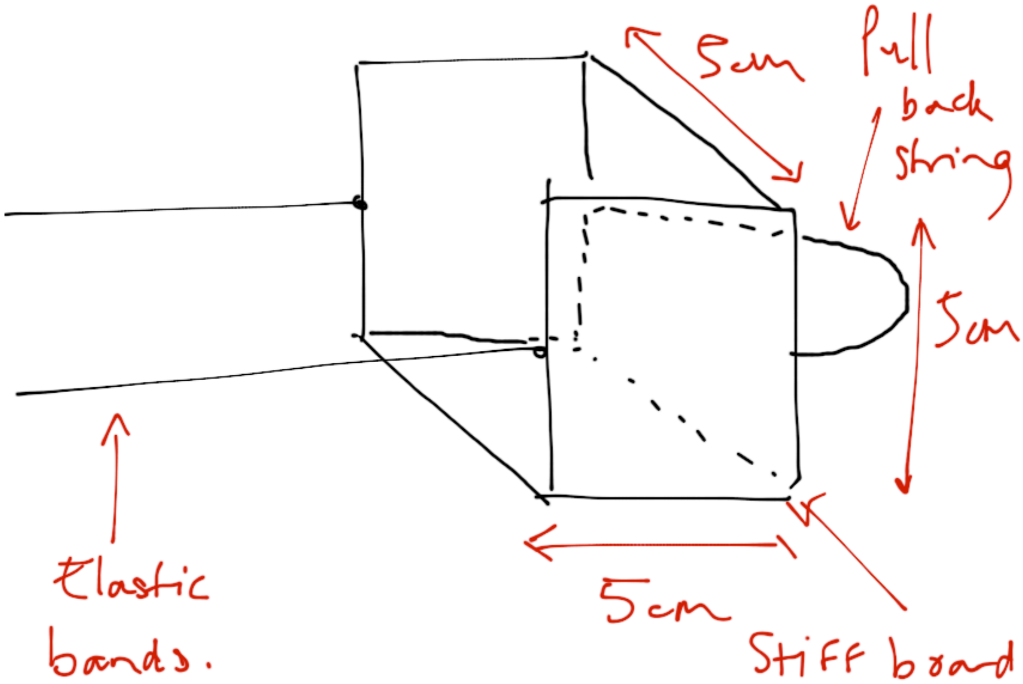


Figure 4
Explanation of angle deviation effect
(Lee, 2012)

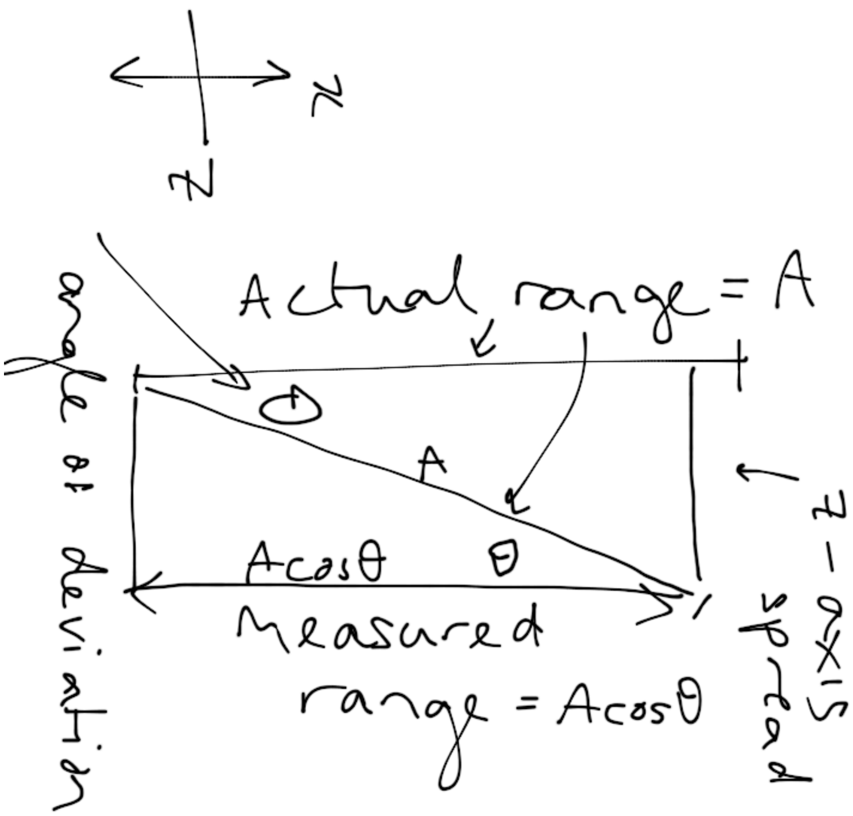


Figure 5
Explanation of why angle deviation effect does not matter
(Lee, 2012)

$$f(x) = \sum_{n=0}^{\infty} \frac{d^{(n)}}{dx} f(x) \Big]_{x_0} \frac{(x-x_0)^n}{n!}$$

For $f(x) = \cos(x)$,

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \dots$$

$\therefore$ As $x \to 0$ (where x = angle deviation)
$\cos(x) \to 1$ ($\frac{x^2}{2!}, \frac{x^4}{4!}, \frac{x^6}{6!} \dots$ are very small for small x)

$\therefore$ A small x has minimal impact on $\cos(x)$ so Z deviation does not affect range measurement much.

Figure 6
Graph of Table Tennis average range against sin(x)
(Lee, Krish and Bain, 2012)

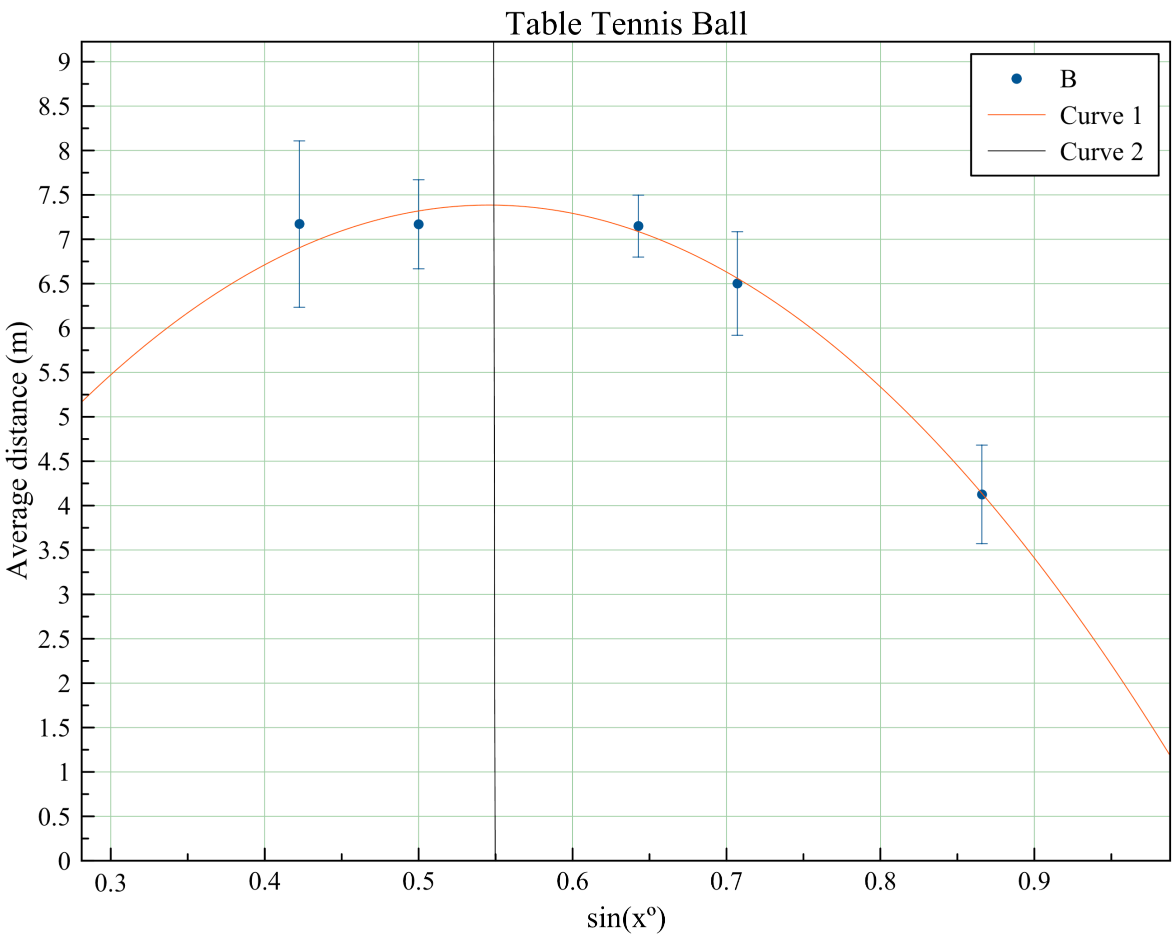
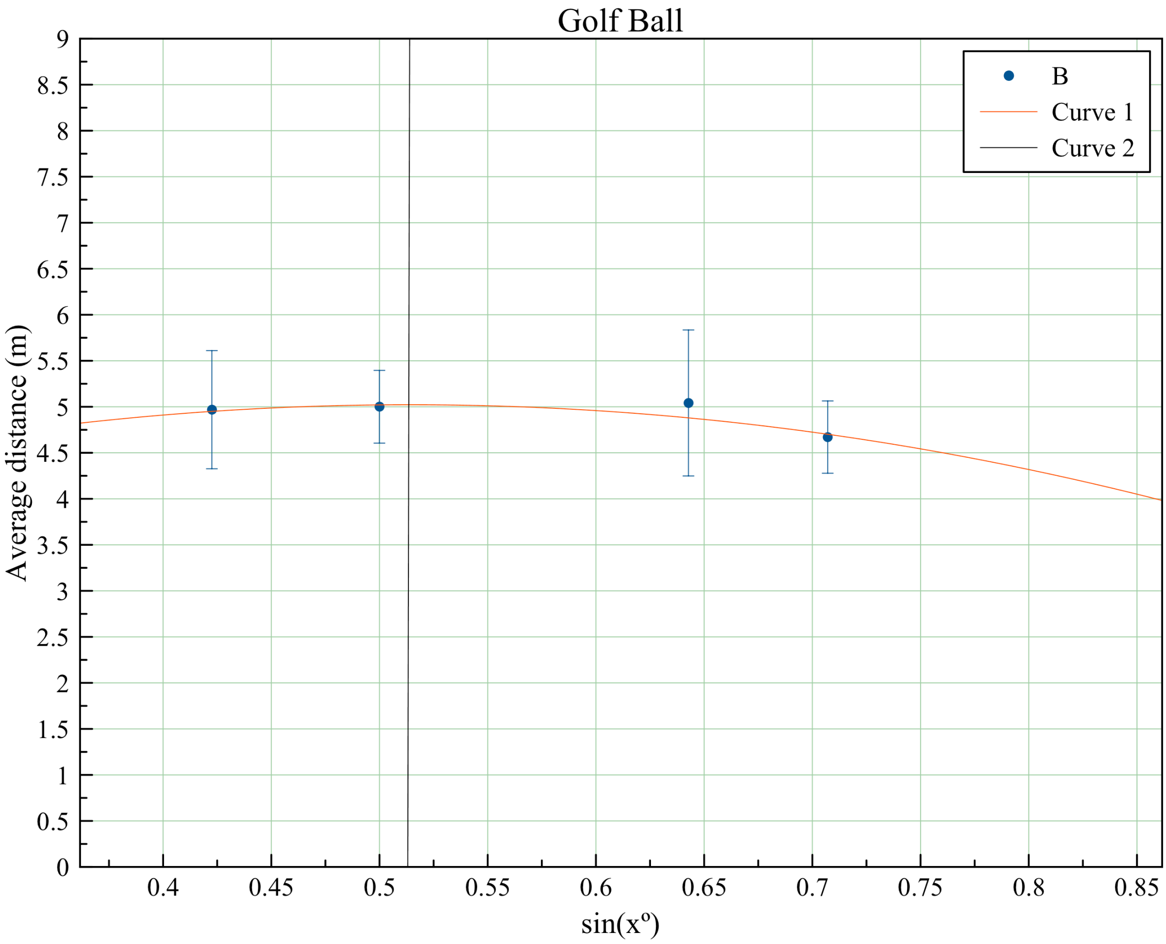


Figure 7
Graph of Golf Ball average range against sin(x)
(Lee, Krish and Bain, 2012)



Bibliography

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Tables

Table 1 - Raw data from Table Tennis ball

Table Tennis		Trial no.										
(x°)	Sin(x°)	1	2	3	4	5	6	7	8	Avg. dist. (m)	Std Dev (m)	Range (m)
25	**0.4226**	7.52	7.69	6.99	7.03	7.16	7.21	6.69	7.09	7.1725	0.3124	1.00
30	**0.5000**	7.30	7.01	7.37	7.22	7.35	6.96	6.99	7.15	7.1688	0.1666	0.41
40	**0.6428**	7.15	7.15	7.28	7.10	7.19	7.16	6.90	7.26	7.1488	0.1169	0.38
45	**0.7071**	6.73	6.28	6.34	6.37	6.60	6.34	6.58	6.78	6.5025	0.1943	0.50
60	**0.8660**	4.02	4.06	3.80	4.31	4.04	4.27	4.14	4.36	4.125	0.1847	0.56

Table 2 - Raw data from Golf ball

Golf		Trial no.										
(x°)	Sin(x°)	1	2	3	4	5	6	7	8	Avg. dist. (m)	Std Dev (m)	Range (m)
25	**0.4226**	5.31	4.77	4.84	5.03	4.79	5.25	4.79	4.97	4.9688	0.2136	0.54
30	**0.5000**	4.99	5.18	5.22	5.00	4.89	4.88	4.96	4.89	5.0013	0.1315	0.34
40	**0.6428**	4.98	5.23	4.88	4.92	5.19	5.55	4.76	4.82	5.0413	0.2643	0.79
45	**0.7071**	4.54	4.82	4.62	4.88	4.53	4.75	4.64	4.58	4.6700	0.1315	0.35
60	**0.8660**	4.07	3.90	3.87	3.89	3.80	4.15	3.93	4.06	3.9588	0.1203	0.35

Table 3 - Data inputed for Fig. 6 (Table Tennis average distance against sin(x), uncertainty of 3 std dev in range)

(x°)	Sin(x°)	Average distance (m)	3 Std Dev (m)	Std Dev (m)
25	**0.4226**	4.9688	0.6407	0.2136
30	**0.5000**	5.0013	0.3946	0.1315
40	**0.6428**	5.0413	0.7930	0.2643
45	**0.7071**	4.6700	0.3944	0.1315
60	**0.8660**	3.9588	0.3609	0.1203

Table 4 - Data inputed for Fig. 7 (Golf ball average distance against sin(x), uncertainty of 3 std dev in range)

(x°)	Sin(x°)	Average (m)	3 Std Dev (m)	Std Dev (m)
25	**0.4226**	7.1725	0.9373	0.3124
30	**0.5000**	7.16875	0.4998	0.1666
40	**0.6428**	7.14875	0.3508	0.1169
45	**0.7071**	6.5025	0.5828	0.1943
60	**0.8660**	4.125	0.5541	0.1847